
SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

Lessons 1 – 2

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1. In general, what is the goal of solving an equation? What does the solution tell you?
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2. Consider the different types of equations you have solved in this unit when answering questions (a) and (b).
 - 2.a. What are the possible types of solutions for a linear equation with one variable?

 - 2.b. What type of equation produces each type of solution you discussed in part (a)?
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3. Sometimes you must simplify expressions as you solve equations. Provide an example of two expressions that are equivalent but not the same. Explain how you know the expressions are equivalent.

Name _____

4. Solve the following equation semi-concretely (with a drawing) and algebraically. Provide justification for each step.

$$3x + 5 = 14$$

SEMI-CONCRETE REPRESENTATION	PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION

Name _____

5. Solve the following equation semi-concretely (with a drawing) and algebraically. Provide justification for each step.

$$2(x + 3) = 2x + 3$$

SEMI-CONCRETE REPRESENTATION	PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION

Name _____

6. Solve the following equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$5(2x - 7) = -10$$

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION

7. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$2(1 + 3x) = 3(2x - 1)$$

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION

8. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$10\left(\frac{1}{2}x - 1\right) = 6x - 10 - x$$

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION

9. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$\frac{1}{4}(x + 8) = 3\left(\frac{1}{4}x + 2\right)$$

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 2

1. In general, what is the goal of solving an equation? What does the solution tell you?

CORRECT ANSWER

The goal of solving an equation is to determine, if possible, the value (s) of the variable that satisfies the equation. The solution tells us the number that, when substituted for the variable, produces a true statement.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The goal is to find x , which tells me what x equals.	thinks the goal is to find what the variable equals and does not relate the variable value back to the original equation	EXPLAIN SOLUTION TO AN EQUATION

2. Consider the different types of equations you have solved in this unit when answering questions (a) and (b).

- 2.a. What are the possible types of solutions for a linear equation with one variable?

CORRECT ANSWER

There are three types of solutions. There could be only one solution or possible value for the variable. There may be no solution because there is no real number that, when substituted for the variable, makes the equation true. Or, there may be infinite solutions because the variable is able to equal any real number.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student states there can be only one solution to a linear equation.	does not understand equations can be identities or contradictions	EXPLAIN <i>CONTRADICTION EQUATION</i> and EXPLAIN <i>IDENTITY EQUATION</i>
The student identifies a specific number of multiple solutions as possible (e.g., two solutions, three solutions, etc.).	does not recognize that if a linear equation has a solution, it will either have one solution or infinitely many solutions	EXPLAIN <i>IDENTITY EQUATION</i>

2.b. What names do we use to describe each type of equation that produces the solutions you discussed in part (a)?

 CORRECT ANSWER

If there is only one solution (or one possible value for the variable), then it is a conditional equation. If there is no solution (or no real number that, when substituted for the variable, makes the equation true), then the equation is a contradiction. If there are infinite solutions (or the variable can equal any real number), then the equation is an identity.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student explains the possible numbers of solutions, but does not know the names of the different types of equations.	understands the types of solutions that are possible, but is lacking the vocabulary to describe the types of equations with the number of solutions	EXPLAIN <i>CONDITIONAL EQUATION</i> , EXPLAIN <i>IDENTITY EQUATION</i> , and EXPLAIN <i>CONTRADICTION EQUATION</i>

3. Sometimes you must simplify expressions as you solve equations. Provide an example of two expressions that are equivalent but not the same. Explain how you know the expressions are equivalent.

CORRECT ANSWER

Answers will vary.

One possible response is:

$9(x - 2)$ and $9x - 18$. I know these expressions are equivalent because I used the distributive property of equality to rewrite the first expression as the second. Also, substituting any value of x in both equations will produce the same result.

$-2(4 - x) = 2x - 8$. I know these expressions are equivalent because when I plug in any number for x , I get the same value for both expressions. For example, if $x = 10$, both expressions equal 12.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student provides two expressions that are not equivalent.	does not understand which operations produce an equivalent expression (e.g., distributing, multiplying by a factor equivalent to one, etc.)	USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS
The student provides two equivalent expressions but cannot explain how they know they are equivalent.	can perform operations to produce equivalent expressions, but cannot explain what makes the expressions equivalent	EXPLAIN EQUIVALENT ALGEBRAIC EXPRESSIONS

4. Solve the equation below semi-concretely (with a drawing) and algebraically. Provide justification for each step.

$$3x + 5 = 14$$

CORRECT ANSWER

SEMI-CONCRETE REPRESENTATION	PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION
	Given Equation	$3x + 5 = 14$
	Subtraction Property of Equality	$-5 \quad -5$ $3x = 9$
	Division Property of Equality	$\frac{3}{3}x = \frac{9}{3}$
		$x = 3$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student incorrectly represents or solves the semi-concrete representation of the equation.	cannot translate an algebraic equation to a semi-concrete model	SOLVE SIMPLE EQUATIONS USING MODELS AND THE CONCEPT OF EQUALITY

Example Error	Misconception	Missing Knowledge
The student gives incorrect Properties of Equality or is unable to state a Property of Equality.	can follow a solving process but cannot justify why the operations they use to solve maintain a balanced equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student adds 5 to both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student multiplies both sides of the equation by 3.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION

5. Solve the equation below semi-concretely (with a drawing) and algebraically. Provide justification for each step.

$$2(x + 3) = 2x + 3$$

CORRECT ANSWER

SEMI-CONCRETE REPRESENTATION	PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION
	Given Equation	$2(x + 3) = 2x + 3$
	Distributive Property of Equality	$2x + 6 = 2x + 3$
	Subtraction Property of Equality	$-2x \quad -2x$ $6 = 3$
	Subtraction Property of Equality	$-3 \quad -3$ $3 = 0$
		no real number, substituted for x, makes the equation true

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student incorrectly represents or solves the semi-concrete representation of the equation.	cannot translate an algebraic equation to a semi-concrete model	SOLVE SIMPLE EQUATIONS USING MODELS AND THE CONCEPT OF EQUALITY
The student gives incorrect Properties of Equality or is unable to state a Property of Equality.	can follow a solving process but cannot justify why the operations they use to solve maintain a balanced equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student does not distribute correctly (e.g., simplifies $2(x + 3)$ as $2x + 3$ instead of $2x + 6$).	does not multiply all terms of an expression in parentheses by the given factor	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
The student adds $2x$ to both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student adds 3 or 6 to both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student does not state that “no real number, substituted for x , makes the equation true.”	is unable to interpret the number of solutions of a linear equation when they get to an equation without a variable	DETERMINE THE NUMBER OF SOLUTIONS OF A 1-VARIABLE LINEAR EQUATION

6. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$5(2x - 7) = -10$$

CORRECT ANSWER

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION
Given Equation	$5(2x - 7) = -10$
Distributive Property of Equality	$10x - 35 = -10$
Addition Property of Equality	$+35 \quad +35$ $10x = 25$
Division Property of Equality	$\frac{10}{10}x = \frac{25}{10}$ $x = \frac{5}{2}$ or $x = 2.5$

This is a conditional equation, which means there is one solution. The only variable value that makes the equation true is $\frac{5}{2}$ or 2.5.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student gives incorrect Properties of Equality or is unable to state a Property of Equality.	can follow a solving process but cannot justify why the operations they use to solve maintain a balanced equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student does not distribute correctly (e.g., simplifies $5(2x - 7)$ as $10x - 7$ instead of $10x - 35$).	does not multiply all terms of an expression in parentheses by the given factor	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
The student subtracts 35 (or another value such as 7 if they made the mistake described in the row above) from both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student multiplies both sides of the equation by 10.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student is unable to explain what their solution means in terms of the variable value in the original equation.	can solve for a variable but is unable to describe what their result means in terms of the variable value in the original equation	EXPLAIN SOLUTION TO AN EQUATION
The student is unable to name or describe the type of equation as conditional.	does not know the different types of equations	EXPLAIN CONDITIONAL EQUATION

7. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$2(1 + 3x) = 3(2x - 1)$$

CORRECT ANSWER

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION
Given Equation	$2(1 + 3x) = 3(2x - 1)$
Distributive Property of Equality	$2 + 6x = 6x - 3$
Subtraction Property of Equality	$-6x \quad -6x$ $2 = -3$
Addition Property of Equality	$+3 \quad +3$ $5 = 0$

This is a contradiction (equation), which means there is no solution, or no real number that, when substituted for x , makes the equation true.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student gives incorrect Properties of Equality or is unable to state a Property of Equality.	can follow a solving process but cannot justify why the operations they use to solve maintain a balanced equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student does not distribute correctly (e.g., simplifies $3(2x - 1)$ as $6x - 1$ instead of $6x - 3$).	does not multiply all terms of an expression in parentheses by the given factor	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
The student adds $6x$ to both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student adds 2 or subtracts 3 from both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student is unable to explain what their solution means in terms of the variable value in the original equation.	can perform operations appropriately but is unable to interpret what their result means in terms of the variable value in the original equation	DETERMINE THE NUMBER OF SOLUTIONS OF A 1-VARIABLE LINEAR EQUATION
The student is unable to name or describe the type of equation as a contradiction.	does not know the different types of equations	EXPLAIN CONTRADICTION EQUATION

8. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$10\left(\frac{1}{2}x - 1\right) = 6x - 10 - x$$

CORRECT ANSWER

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION
Given Equation	$10\left(\frac{1}{2}x - 1\right) = 6x - 10 - x$
Distributive Property of Equality	$5x - 10 = 5x - 10$
Subtraction Property of Equality	$-5x \quad -5x$
	$-10 = -10$
Addition Property of Equality	$+10 \quad +10$
	$0 = 0$
	x can equal any real number

This is an identity (equation), which means there are infinite solutions, or x can equal any real number and the equation will be true.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student gives incorrect Properties of Equality or is unable to state a Property of Equality.	can follow a solving process but cannot justify why the operations they use to solve maintain a balanced equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student does not distribute correctly (e.g., simplifies $10\left(\frac{1}{2}x - 1\right)$ as $5x - 1$ instead of $5x - 10$).	does not multiply all terms of an expression in parentheses by the given factor	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
The student does not combine like terms correctly (e.g., simplifies $6x$ and $-x$ as $7x$ instead of $5x$).	does not keep the negative sign with the variable it precedes	USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS
The student adds $5x$ to both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student subtracts 10 from both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student is unable to explain what their solution means in terms of the variable value in the original equation.	can perform operations appropriately, but is unable to interpret what their result means in terms of the variable value in the original equation	DETERMINE THE NUMBER OF SOLUTIONS OF A 1-VARIABLE LINEAR EQUATION
The student is unable to name or describe the type of equation as an identity.	does not know the different types of equations	EXPLAIN <i>IDENTITY EQUATION</i>

9. Solve the equation algebraically and provide justification for each operation. Then determine if the equation is conditional, an identity, or a contradiction, and explain how this relates to the variable value in the original equation.

$$\frac{1}{4}(x + 8) = 3\left(\frac{1}{4}x + 2\right)$$

CORRECT ANSWER

PROPERTY OF EQUALITY	ALGEBRAIC REPRESENTATION
Given Equation	$\frac{1}{4}(x + 8) = 3\left(\frac{1}{4}x + 2\right)$
Distributive Property of Equality	$\frac{1}{4}x + 2 = \frac{3}{4}x + 6$
Subtraction Property of Equality	$-\frac{1}{4}x \quad -\frac{1}{4}x$
Subtraction Property of Equality	$2 = \frac{1}{2}x + 6$
Subtraction Property of Equality	$-6 \quad -6$
Subtraction Property of Equality	$-4 = \frac{1}{2}x$
Multiplication Property of Equality	$2(-4) = 2\left(\frac{1}{2}x\right)$
	$-8 = x$

This is a conditional equation, which means there is one solution. The only value that makes the equation true is -8.

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student gives incorrect Properties of Equality or is unable to state a Property of Equality.	can follow a solving process but cannot justify why the operations they use maintain a balanced equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student does not distribute correctly (e.g., simplifies $3(\frac{1}{4}x + 2)$ as $\frac{3}{4}x + 2$ or $\frac{1}{12}x + 6$ instead of $\frac{3}{4}x + 6$).	does not multiply all terms of an expression in parentheses by the given factor or incorrectly multiplies by a fraction	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
The student adds $\frac{1}{4}x$ or $\frac{3}{4}x$ to both sides of the equation.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student multiplies both sides of the equation by $\frac{1}{2}$.	does not understand how to use inverse operations to solve an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student is unable to explain what their solution means in terms of the variable value in the original equation.	can solve for a variable but is unable to describe what their result means in terms of the variable value in the original equation	EXPLAIN SOLUTION TO AN EQUATION
The student is unable to name or describe the type of equation as conditional.	does not know the different types of equations	EXPLAIN CONDITIONAL EQUATION