**RATIONAL NUMBER ADDITION AND SUBTRACTION**

**INSTRUCTIONAL ACTIVITY**

*Lesson 4*

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**LEARNING GOAL**

Students will extend their understanding of integer addition and subtraction to addition and subtraction of rational numbers.

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**PRIMARY ACTIVITY**

Students will use their existing understanding of adding and subtracting integers formed in the credit and debit activity to add and subtract rational numbers. Students may utilize strategies from the credit and debt activity to reason through rational number addition and subtraction.

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**OTHER VOCABULARY**

Students will need to know the meaning of the following terms:

- Number line
- Integers
- Rational numbers
- Sum

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**MATERIALS**

- Instructional Activity Student Handout
- Instructional Activity Supplement *(Recommend one copy for every student.)*

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**IMPLEMENTATION**

Students should reference their knowledge of adding and subtracting integers from *Lesson 3* as they extend their understandings of integer addition and subtraction to all rational numbers.

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Review the scaffold students used to add and subtract integers in Lesson 3. Following is an example provided in the Instructional Activity Student Handout for Lesson 3.

<table>
<thead>
<tr>
<th>Expression:</th>
<th>3 + (-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting value/balance:</td>
<td>3</td>
</tr>
<tr>
<td>Operation meaning (circle one):</td>
<td>add to the balance</td>
</tr>
<tr>
<td>Value of the change:</td>
<td>-5</td>
</tr>
<tr>
<td>In the end, the balance will (circle one):</td>
<td>increase</td>
</tr>
<tr>
<td>Alternate expression (if applicable):</td>
<td>3 – 5</td>
</tr>
<tr>
<td>Number line representation:</td>
<td><img src="image" alt="Number Line" /></td>
</tr>
<tr>
<td>Result:</td>
<td>-2</td>
</tr>
</tbody>
</table>

Ask students if they can think of any other types of numbers that can be positive or negative. Guide students to consider fractions and decimals, which they should have experience with from earlier grades.

Discuss how students can extend their understandings of addition and subtraction of integers to fractions and decimals (rational numbers).

Review, as needed, addition and subtraction of positive fractions and decimals.

Extend students’ knowledge of adding and subtracting positive fractions to include negative fractions, beginning with the worked out example provided in the Instructional Activity Student Handout.

Students should consider and operate with fractions on a number line in a way that is consistent with their work with integers.

NOTE: When adding and subtracting fractions on a number line, once students determine the least common denominator for the fractions they are adding or subtracting, they should eventually use this value to partition the number line. For example, if the least common denominator is 12, students may start by partitioning the number line into fourths because one denominator is 4, but should then continue to partition the number line into twelfths. An awareness of the value of the fraction and whether the initial value will increase or decrease as a result of the operation will help students determine how much and which portion of the number line must be labeled to model the addition or subtraction. A scaffold for students who struggle to set up the number line would be for the teacher to guide the process of setting up the number line for them, or provide a pre-labeled number line, to focus their attention on concepts of addition and subtraction of rational numbers.

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Provide a few additional examples of addition and subtraction expressions containing negative fractions. Start with examples that can be modeled on a number line labeled from negative two to positive two, then move to examples that require students to draw their own number line and consider the values that must be present to model the expression.

Review the idea that the unit length must remain constant on the number line, even when it is subdivided to accommodate fractions or decimals. Students may have a tendency to make more space for subdivided units than units that are not subdivided.

Require students to model the addition or subtraction on the number line provided in the Instructional Activity Supplement.

Relate the symbols back to the credit and debit activity with chores from Lesson 1 and operations with fractions from previous instruction.

Once students show an understanding of addition and subtraction when the expression contains negative fractions, shift students’ focus to operations with decimals.

NOTE: Adding and subtracting decimals can be modeled with the concept of credits and debits alone or in conjunction with the number line. Solid number sense, additive inverses, the commutative property, and the use of the number line, even if it is an approximation, will allow students to reason through operations with decimals without using a calculator. For example, in the expression 1.2 + (−2.35), in order to operate by hand, it is beneficial to decompose −2.35 into −1.2 and −1.15 so students are able to see that the zero pair, 1.2 and −1.2, simplifies to zero and, therefore, −1.15 is the simplest form of the expression. This should be approximately the resulting location on the number line if a student plots a point at 1.2 and models a decrease of 2.35. When possible, appealing to students’ natural tendency to combine negative numbers in the same way as positive numbers is beneficial as well.

Provide a few additional examples of addition and subtraction containing negative decimals to discuss as a class. Start with examples that can be modeled on a number line labeled from negative two to positive two, then move to examples that require students to draw their own number line and consider the values that must be present to accommodate the expression.

Require students to model the addition or subtraction on the number line provided in the Instructional Activity Supplement. Students should be precise as they work with decimals to the tenths place, but may need to plot the initial value and then approximate with decimals to the hundredths place or beyond.

Relate the symbols back to the credit and debit activity with chores from Lesson 1 and operations with decimals from previous instruction.

Students should now be ready to complete the Instructional Activity Student Handout in pairs or small groups. Blank number lines are provided for all questions, though it is reasonable to...
only require students to operate precisely on number lines when operating with fractions and decimals to the tenths place. When expressions include decimals to the hundredths place or beyond, students should accurately place the initial value, then approximate the increase or decrease on the number line in order to estimate the simplified value, which must be calculated by hand for precision. Modeling precisely on the number line serves to enhance students’ representational fluency and strengthen the student’s conceptual framework for working with signed numbers. Ultimately, students should think abstractly about relative positions on a number line without needing a concrete model to simplify every addition and subtraction expression. An estimation of the number line along with computation is the goal for most students.

GUIDING QUESTIONS

Elicit student thinking:

- Does adding always increase the value/balance?
- Does subtracting always decrease the value/balance?
- How is adding and subtracting fractions on a number line similar to adding and subtracting integers? How is it different?

Determine if the student can ADD RATIONAL NUMBERS:

- What is the original value?
- Are you adding or removing a value?
- What value is being added to the original value/balance?
- Will the original value increase or decrease as a result?
- What is the result of the addition?
- How can you describe this situation in terms of credits and debits?
Determine if the student can REPRESENT ADDITION OF RATIONAL NUMBERS ON A NUMBER LINE:

- What is the initial value? Where is this value located on the number line?
- Are you adding or removing a value?
- What value is being added to the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you model this on a number line? What intervals would help you model the addition?
- What is the result of the addition?

Determine if the student can EXPLAIN ADDITION OF RATIONAL NUMBERS:

- What is the initial value?
- Are you adding or removing a value?
- What value is being added to the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you describe this situation in terms of credits and debits?
- How would addition be represented on a number line?

Determine if the student can SUBTRACT RATIONAL NUMBERS:

- What is the initial value?
- Are you adding or removing a value?
- What value is being removed from the initial value/balance?
- Will the original value increase or decrease as a result?
- What is the result of the subtraction?
- How can you describe this situation in terms of credits and debits?
Determine if the student can **REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE**:

- What is the initial value? Where is this value located on the number line?
- Are you adding or removing a value?
- What value is being removed from the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you model this on a number line? What intervals would help you model the subtraction?
- What is the result of the subtraction?

Determine if the student can **EXPLAIN SUBTRACTION OF RATIONAL NUMBERS**:

- What is the initial value?
- Are you adding or removing a value?
- What value is being removed from the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you describe this situation in terms of credits and debits?
- How would subtraction be represented on a number line?

Determine if the student can **APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS**:

- [Provide students with an expression containing addition of two rational numbers.] Can you demonstrate this addition either with credits and debits or on the number line? What happens to the sum if the numbers are switched? Does the sum change?
Determine if the student can **EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS:**

- Can you explain this addition in terms of credits and debits?
- Can you think of a context, other than credits and debits, that this addition could describe?

Determine if the student can **EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS:**

- Can you explain this subtraction in terms of credits and debits?
- Can you think of a context, other than credits and debits, that this subtraction could describe?

Students should be required to thoroughly complete each question in the **INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.** If students are struggling to do so, they should utilize scaffolds from the credit and debit activity in **LESSON 1.** Sketching a piggy bank and individual credits and debits may be beneficial for some students.

At the end of the activity, provide students with a variety of additional rational number addition and subtraction questions without the written scaffolds to guide their thinking. Include examples of subtraction that require students to regroup. Require students to use whatever means necessary to simplify each expression.
Rational Number Addition and Subtraction

Lesson 4

Use the following structure to simplify each expression. Thinking in terms of the balance and adjusting credits and debits may be beneficial. The first question has been completed for you as an example. Note that there may not be an equivalent expression for all questions (e.g., $2 + 1$ is the most simplified form of the expression).

1. Expression: $\frac{-3}{4} - \left(-\frac{5}{2}\right)$

Starting value/balance: $\frac{-3}{4}$

Operation meaning (circle one): add to the balance

Value of the change: $\frac{-5}{2}$

In the end, the balance will (circle one): increase

Alternate expression (if applicable): $\frac{-3}{4} + \frac{10}{4}$

Number line representation:

Result: $\frac{7}{4}$

2. Expression: $1.4 + (-2.6)$

Starting value/balance: ______

Operation meaning (circle one): add to the balance

Value of the change: ______

In the end, the balance will (circle one): increase

Alternate expression (if applicable): ________

Number line representation:

Result:
3. Expression: \[ \frac{2}{3} - \frac{7}{6} \]
Starting value/balance: 
Operation meaning (circle one): add to the balance subtract from the balance
Value of the change: 
In the end, the balance will (circle one): increase decrease
Alternate expression (if applicable): 
Number line representation:

\[ \begin{array}{c}
-2 \\
-1 \\
0 \\
1 \\
2 \\
\end{array} \]

Result:

4. Expression: \[ -\frac{7}{5} + \frac{1}{2} \]
Starting value/balance: 
Operation meaning (circle one): add to the balance subtract from the balance
Value of the change: 
In the end, the balance will (circle one): increase decrease
Alternate expression (if applicable): 
Number line representation:

\[ \begin{array}{c}
-2 \\
-1 \\
0 \\
1 \\
2 \\
\end{array} \]

Result:

5. Expression: \[ -5.13 - (-1.47) \]
Starting value/balance: 
Operation meaning (circle one): add to the balance subtract from the balance
Value of the change: 
In the end, the balance will (circle one): increase decrease
Alternate expression (if applicable): 
Number line representation (approximate):

\[ \begin{array}{c}
-2 \\
-1 \\
0 \\
1 \\
2 \\
\end{array} \]

Result:
6. Expression: \(-\frac{3}{2} - \frac{5}{6}\)

Starting value/balance: 

Operation meaning (circle one): add to the balance subtract from the balance

Value of the change: 

In the end, the balance will (circle one): increase decrease

Alternate expression (if applicable): 

Number line representation:

Result:

7. Expression: \(13.1 - (-4.55)\)

Starting value/balance: 

Operation meaning (circle one): add to the balance subtract from the balance

Value of the change: 

In the end, the balance will (circle one): increase decrease

Alternate expression (if applicable): 

Number line representation (approximate):

Result:

8. Expression: \(\frac{3}{8} + \frac{3}{2}\)

Starting value/balance: 

Operation meaning (circle one): add to the balance subtract from the balance

Value of the change: 

In the end, the balance will (circle one): increase decrease

Alternate expression (if applicable): 

Number line representation:

Result:
9. Expression: \[ -2.39 + (-1.47) \]

Starting value/balance: _____

Operation meaning (circle one): add to the balance subtract from the balance

Value of the change: _____

In the end, the balance will (circle one): increase decrease

Alternate expression (if applicable): __________

Number line representation (approximate):

Result:
RATIONAL NUMBER ADDITION AND SUBTRACTION

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 4